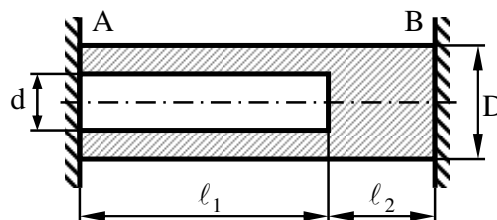


1.)

A furatos henger szobahőmérsékleten hézag- és erőmentesen illeszkedik a megtámasztások közé. Mekkora reakcióerők ébrednek, ha a hőmérsékletet $\Delta t = 30^\circ\text{C}$ -kal megnöveljük? Mekkora a feszültség az egyes szakaszokban?



Adatok: $D = 10\text{ mm}$, $d = 5\text{ mm}$, $l_1 = 10\text{ cm}$, $l_2 = 5\text{ cm}$, $\alpha = 1,2 \cdot 10^{-5} \frac{1}{^\circ\text{C}}$, $E = 210\text{ GPa}$

2.)

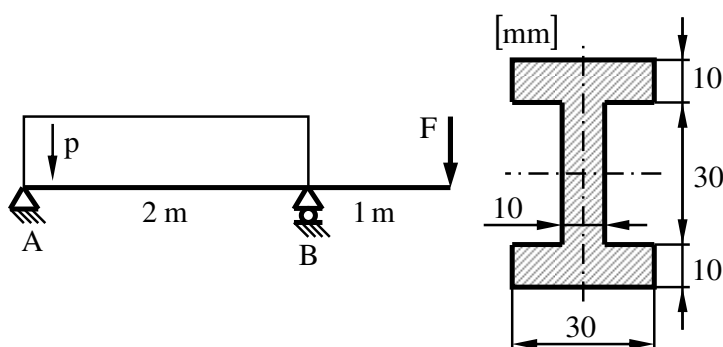
Ellenőrizze a tartó kritikus keresztmetszetét szilárdsági szempontból (hajlításra)!

Adatok:

$$p = 1,5\text{ kN/m}$$

$$F = 1\text{ kN}$$

$$\sigma_{\text{meg}} = 120\text{ MPa}$$



3.)

Adja meg a semleges tengely és az x tengely által bezárt szöget! Ábrázolja is a semleges tengely elhelyezkedését! Számítsa ki a P pontban ébredő feszültség értékét! Húzó- vagy nyomófeszültségről van szó?

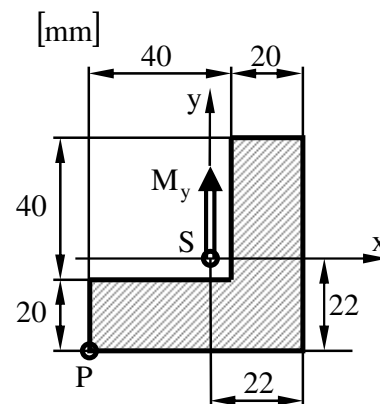
Az S súlypont helyzete és a másodrendű nyomatékok adottak.

Adatok:

$$M = 1\text{ kNm}$$

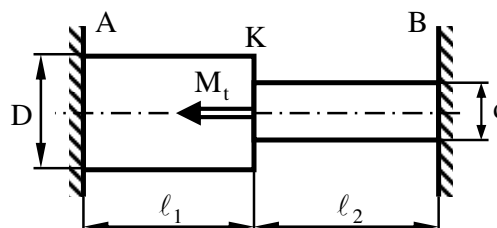
$$I_x = I_y = 578667\text{ mm}^4$$

$$I_{xy} = 288000\text{ mm}^4$$



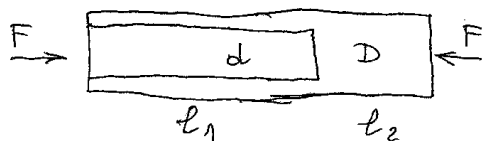
4.)

A változó keresztmetszetű hengeres alkatrész mindkét végén befogott. Számítsa ki a reakciónyomatékokat! Mekkora a K keresztmetszet elfordulása?



Adatok: $D = 12\text{ mm}$, $d = 6\text{ mm}$, $l_1 = 5\text{ cm}$, $l_2 = 7\text{ cm}$, $G = 80\text{ GPa}$, $M_t = 20\text{ Nm}$

B1



$$\begin{aligned}
 D &= 10 \text{ mm} & \alpha &= 11,2 \cdot 10^{-5} \frac{1}{^\circ\text{C}} \\
 d &= 5 \text{ mm} & E &= 210 \text{ GPa} \\
 l_1 &= 10 \text{ cm} & F &=? \\
 l_2 &= 5 \text{ cm} & \sigma_1 &=? \\
 \Delta t &= 30^\circ\text{C} & \sigma_2 &=?
 \end{aligned}$$

$$A_1 = \frac{(D^2 - d^2)\pi}{4} = \frac{(10^2 - 5^2)\pi}{4} = 58,90 \text{ mm}^2$$

$$A_2 = \frac{D^2\pi}{4} = \frac{10^2\pi}{4} = 78,54 \text{ mm}^2$$

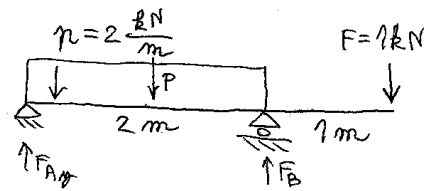
$$\alpha(l_1 + l_2)\Delta t = \frac{F l_1}{A_1 E} + \frac{F l_2}{A_2 E}$$

$$F = \frac{\alpha E (l_1 + l_2) \Delta t}{\frac{l_1}{A_1} + \frac{l_2}{A_2}} = \frac{11,2 \cdot 10^{-5} \cdot 210 \cdot 10^3 (100 + 50) \cdot 30}{\frac{100}{58,9} + \frac{50}{78,54}} = 4858 \text{ N (nyg)}$$

$$\sigma_1 = \frac{N_1}{A_1} = \frac{-F}{A_1} = \frac{-4858}{58,9} = -82,48 \text{ MPa (nyg)}$$

$$\sigma_2 = \frac{N_2}{A_2} = \frac{-F}{A_2} = \frac{-4858}{78,54} = -61,85 \text{ MPa (nyg)}$$

B2



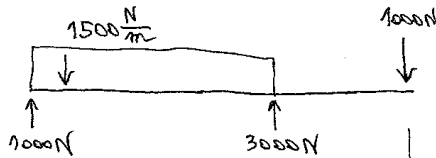
$$\sigma_{meg} = 120 \text{ MPa}$$

$$\sum M_A = 0 = -P \cdot 1 + F_B \cdot 2 - F \cdot 3$$

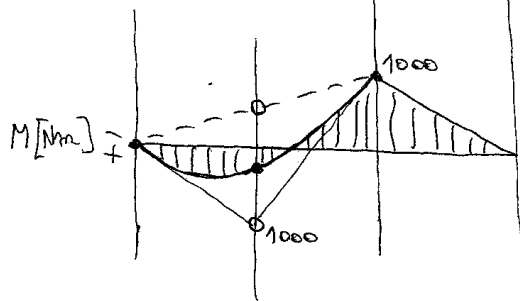
$$F_B = \frac{P + 3F}{2} = \frac{3 + 3 \cdot 1}{2} = 3 \text{ kN} (\uparrow)$$

$$\sum F_y = 0 = F_{Ay} - P + F_B - F$$

$$F_{Ay} = P - F_B + F = 3 - 3 + 1 = 1 \text{ kN} (\uparrow)$$



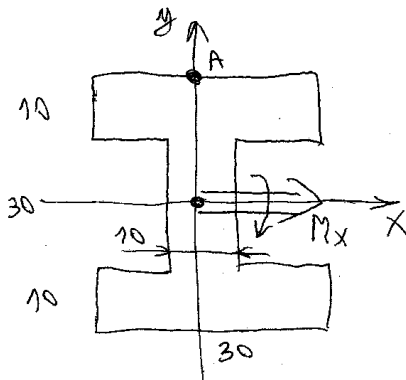
$$M_{max} = 1 \text{ kNm}$$



$$M_x = +M_{max} = 1 \text{ kNm}$$

$$I_x = \frac{50^3 \cdot 30}{12} - \frac{30^3 \cdot 20}{12} =$$

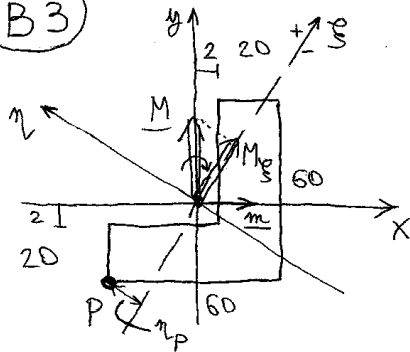
$$= 312500 - 45000 = 267500 \text{ mm}^4$$



$$\sigma_{max} = \sigma_A = \frac{M_x}{I_x} y_A = \frac{10^6}{267500} (+25) = 93,46 \text{ MPa}$$

$$\sigma_{max} < \sigma_{meg} \quad \checkmark \text{ OK}$$

B3



$$M = 1 \text{ kNm}$$

$$I_x = I_y = 578\,667 \text{ mm}^4$$

$$I_{xy} = 288\,000 \text{ mm}^4$$

$$\underline{M} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ kNm} \rightarrow \underline{m} \perp \underline{M} \rightarrow \underline{m} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \underline{m} \underline{I}_{xy} \underline{e}_\xi &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 578\,667 & -288\,000 \\ -288\,000 & 578\,667 \end{bmatrix} \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} = \\ &= \begin{bmatrix} 578\,667 & -288\,000 \end{bmatrix} \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} = 578\,667 \cos\beta - 288\,000 \sin\beta = 0 \end{aligned}$$

$$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{578\,667}{288\,000} = 2,009 \rightarrow \boxed{\beta = 63,54^\circ}$$

$$\underline{e}_\xi = \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} = \begin{bmatrix} 0,4456 \\ 0,8952 \end{bmatrix} \rightarrow \underline{e}_\eta = \begin{bmatrix} -0,8952 \\ 0,4456 \end{bmatrix}$$

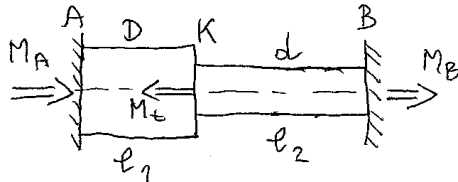
$$\underline{M}_\xi = \underline{e}_\xi^T \underline{M} = \begin{bmatrix} 0,4456 & 0,8952 \end{bmatrix} \begin{bmatrix} 0 \\ 1000 \end{bmatrix} = 895,2 \text{ Nm}$$

$$\underline{\eta}_P = \underline{e}_\eta^T \underline{r}_P = \begin{bmatrix} -0,8952 & 0,4456 \end{bmatrix} \begin{bmatrix} -38 \\ -22 \end{bmatrix} = 24,21 \text{ mm}$$

$$\begin{aligned} \underline{I}_\xi &= \underline{e}_\xi^T \underline{I}_{xy} \underline{e}_\xi = \begin{bmatrix} 0,4456 & 0,8952 \end{bmatrix} \begin{bmatrix} 578\,667 & -288\,000 \\ -288\,000 & 578\,667 \end{bmatrix} \begin{bmatrix} 0,4456 \\ 0,8952 \end{bmatrix} = \\ &= \begin{bmatrix} 0,4456 & 0,8952 \end{bmatrix} \begin{bmatrix} 36,42 \\ 389\,690 \end{bmatrix} = 348\,867 \text{ mm}^4 \end{aligned}$$

$$\boxed{\sigma_P = \frac{M_\xi}{I_\xi} \eta_P = \frac{895,2 \cdot 10^3}{348\,867} (24,21) = +62,12 \text{ MPa (h)}}$$

B4



$$\begin{aligned}
 D &= 12 \text{ mm} & G &= 80 \text{ GPa} \\
 d &= 6 \text{ mm} & M_t &= 20 \text{ Nm} \\
 l_1 &= 5 \text{ cm} & M_A &=? \quad M_B=? \\
 l_2 &= 7 \text{ cm} & \varphi_K &=?
 \end{aligned}$$

$$I_{p1} = \frac{D^4 \pi}{32} = \frac{12^4 \pi}{32} = 2036 \text{ mm}^4$$

$$I_{p2} = \frac{d^4 \pi}{32} = \frac{6^4 \pi}{32} = 127,2 \text{ mm}^4$$

$$1.) \sum M_t = 0 = M_A - M_t + M_B$$

$$2.) \varphi_K = \varphi_1 = \varphi_2 = \frac{M_A l_1}{I_{p1} G} = \frac{M_B l_2}{I_{p2} G}$$

$$1.) M_B = M_t - M_A$$

$$1 \rightarrow 2.) \frac{M_A l_1}{I_{p1} G} = \frac{(M_t - M_A) l_2}{I_{p2} G}$$

$$M_A = \frac{M_t \frac{l_2}{I_{p2}}}{\frac{l_1}{I_{p1}} + \frac{l_2}{I_{p2}}} = \frac{M_t}{\frac{l_1 I_{p2}}{l_2 I_{p1}} + 1} = \frac{20}{\frac{5 \cdot 127,2}{7 \cdot 2036} + 1} = 19,15 \text{ Nm} (\Rightarrow)$$

$$1.) M_B = M_t - M_A = 20 - 19,15 = 0,85 \text{ Nm} (\Rightarrow)$$

$$2.) \varphi_K = \varphi_1 = \frac{M_A l_1}{I_{p1} G} = \frac{19,15 \cdot 10^3 \cdot 50}{2036 \cdot 80 \cdot 10^3} = 5,879 \cdot 10^{-3} \text{ rad} = 0,3368^\circ$$